Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

MATHÉMATIQUES

QUELQUES REMARQUES SUR LE THÉORÉME DE STEKLOFF-SEVERINI

Par I. NATANSON

(Présenté par S. N. Bernstein, de l'Académie, le 31. III. 1936)

Le théorème connu de Stekloff (1)-Severini (2), concernant des systèmes orthogonaux, porte que *chaque système* $\{\omega_n(x)\}$ *fermé dans la classe des polynômes est fermé dans la classe* (L^2) *des fonctions intégrables avec son carré.* Dans cet énoncé on peut remplacer la classe des polynômes par l'ensemble arbitraire $\mathfrak{A}$ des fonctions dense partout dans la classe (L^2).

Si nous posons

$$\sum_{k=1}^{n} \omega_k(t)\,\omega_k(x) = \Phi_n(t, x), \tag{1}$$

alors la fermeture du système $\{\omega_n(x)\}$ est équivalente à la convergence en moyenne de l'intégrale singulière

$$\int_a^b f(t)\,\Phi_n(t, x)\,dt. \tag{2}$$

Donc, le théorème de Stekloff-Severini établit la convergence en moyenne de l'intégrale (2) pour chaque fonction $f(t) \in L^2$, en moyennant sa convergence seulement pour quelques fonctions de cette classe.

Dans cette note je vais considérer la question analogue pour les intégrales singulières du type générale sans supposer que le noyau ait la forme (1).

Lemme. *Soit le noyau* $\Phi(t, x)$ *symétrique et tel que l'intégrale*

$$\int_a^b \int_a^b \Phi^2(t, x)\,dtdx$$

est finie. Si la moindre * *valeur caractéristique du noyau* $\Phi(x, t)$ *(comme du noyau de l'équation intégrale) est* λ_1, *alors pour chaque fonction* $f(t) \in (L^2)$ *on a*

$$\left\| \int_a^b f(t)\,\Phi(t, x)\,dt \right\| \leqslant \frac{1}{|\lambda_1|} \cdot \|f(x)\|, \tag{3}$$

* En module.

où nous posons en général

$$\| h(x) \| = \sqrt{\int_a^b h^2(x)\, dx}.$$

Le nombre $\frac{1}{\lambda_1}$ *ne peut être diminué.*

Démonstration*. En vertu du théorème connu de Hilbert-Schmidt on a:

$$\int_a^b f(t)\, \Phi(t, x)\, dt = \sum_{k=1}^{\infty} \frac{c_k}{\lambda_k} \varphi_k(x),$$

où $\varphi_k(x)$ et λ_k sont des fonctions et des valeurs fondamentales du noyau et c_k sont des coefficients de Fourier de la fonction $f(t)$ par rapport à $\{\varphi_k(x)\}$. L'élevant au carré et l'intégrant, nous trouvons:

$$\left\| \int_a^b f(t)\, \Phi(t, x)\, dt \right\|^2 = \sum_{k=1}^{\infty} \frac{c_k^2}{\lambda_k^2} \leqslant \frac{1}{\lambda_1^2} \sum_{k=1}^{\infty} c_k^2,$$

d'où, en vertu de l'inégalité de Bessel, suit notre lemme. Comme pour $f(x) = \varphi_1(x)$ dans (3) a lieu l'égalité, le nombre $\frac{1}{|\lambda_1|}$ ne peut être diminué.

Remarque. L'inégalité (3) a lieu aussi pour le noyau non symétrique, si λ_1 désigne la moindre valeur caractéristique du noyau de Schmidt

$$\Psi(t, x) = \int_a^b \Phi(z, t)\, \Phi(z, x)\, dz.$$

Théorème. *Soit*

$$\lim_{n \to \infty} \left\| g(x) - \int_a^b g(t)\, \Phi_n(t, x)\, dt \right\| = 0,$$

quelle que soit la fonction $g(t)$ *d'une classe* $\mathfrak{A}$ *dense partout dans l'espace* (L^2). *Considérons la totalité des valeurs caractéristiques du noyau* $\Phi_n(t, x)$ *(si ce noyau est non symétrique, il faut considérer les valeurs caractéristiques du noyau de Schmidt)*:

$$|\lambda_1^{(n)}| \leqslant |\lambda_2^{(n)}| \leqslant |\lambda_3^{(n)}| \leqslant \ldots$$

Pour que l'intégrale (2) *converge en moyenne pour chaque fonction* $f(t) \in (L^2)$, *il faut et il suffit que le zéro ne soit pas un point d'accumulation des nombres* $\{\lambda_k^{(n)}\}$.

Démonstration. Si la condition du théorème est remplie, alors

$$\frac{1}{|\lambda_1^{(n)}|} \leqslant k.$$

Soit $\varepsilon > 0$. Choisissons dans la classe $\mathfrak{A}$ une fonction $g(t)$ telle que

$$\| f(x) - g(x) \| < \varepsilon. \qquad (4)$$

* Aussi ce lemme peut être établi comme une conséquence de la théorie générale des opérations linéaires dans l'espace de Hilbert.

En vertu du lemme on a:

$$\left\| \int_a^b f(t)\,\Phi_n(t,x)\,dt - \int_a^b g(t)\,\Phi_n(t,x)\,dt \right\| \leqslant k\varepsilon, \tag{5}$$

quelle que soit n. Or, pour $n > N$ nous avons:

$$\left\| g(x) - \int_a^b g(t)\,\Phi_n(t,x)\,dt \right\| \leqslant \varepsilon,$$

d'où pour ces n

$$\left\| f(x) - \int_a^b f(t)\,\Phi_n(t,x)\,dt \right\| < \varepsilon(k+2),$$

et la suffisance de la condition est établie.

Sa nécessité est une conséquence d'un théorème général d'analyse fonctionelle, en vertu du quel les normes d'une suite des opérations linéaires partout convergente sont bornées.

Remarque I. La condition du théorème est évidemment, remplie, si

$$\int_a^b \int_a^b \Phi_n^2(t,x)\,dtdx \leqslant k. \tag{6}$$

Mais, déjà pour les noyaux (1) l'inégalite (6) n'est pas valable. En effet, pour ces noyaux-ci nous avons:

$$\int_a^b \int_a^b \Phi_n^2(t,x)\,dtdx = n.$$

Cependant la condition de notre théorème est remplie pour ces noyaux, parce qu'on a $\lambda_1^{(n)} = 1$.

Remarque II. On peut démontrer un théorème analogue concernant *la convergence en mesure*:

1) *Soit* $\int_\alpha^\beta \Phi_n(t,x)\,dt$ *converge en mesure vers* 1 *dans l'intervalle* $\alpha < x < \beta$ *quelle que soit cet intervalle* $(a \leqslant \alpha < \beta \leqslant b)$;

2) *le zéro n'est pas un point d'accumulation des nombres* $\{\lambda_k^{(n)}\}$.

Alors, l'integrale (2) *converge en mesure vers* $f(x)$ *pour chaque fonction* $f(t)$ *e* (L^2).

En effet, en vertu de la condition 1) nous pouvons affirmer la convergence en mesure de l'intégrale (2) pour chaque fonction $g(t)$ pour laquelle l'intervalle (a, b) se divise en nombre fini des intervalles de stationnement. Soit, maintenant, $f(t)$ une fonction arbitraire appartenant à (L^2). Choisissons une fonction $g(t)$, satisfaisant, la relation (4) [et, par conséquent, (5)]. Alors, quel que soit le nombre $\eta > 0$, nous avons:

$$\underset{x}{mE}\left(|f(x) - g(x)| > \eta\right) \leqslant \frac{\varepsilon^2}{\eta^2};$$

$$\underset{x}{mE}\left(\left|\int_a^b f(t)\,\Phi_n(t,x)\,dt - \int_a^b g(t)\,\Phi_n(t,x)\,dt\right| > \eta\right) \leqslant \frac{k^2\varepsilon^2}{\eta^2};$$

car la relation

$$\int_a^b h^2(x)\,dx \leqslant A$$

implique la relation

$$\underset{x}{mE}(|h(x)|>\eta) \leqslant \frac{A}{\eta^2}.$$

Or, pour $n>N$, nous avons:

$$\underset{x}{mE}\left(\left|g(x)-\int_a^b g(t)\,\Phi_n(t,x)\,dt\right|>\eta\right)\leqslant\varepsilon^2,$$

d'où pour ces valeurs de n, nous avons:

$$\underset{x}{mE}\left(\left|f(x)-\int_a^b f(t)\,\Phi_n(t,x)\,dt\right|>3\eta\right)<\varepsilon^2\left(1+\frac{k^2+1}{\eta^2}\right),$$

ce qu'il fallait démontrer.

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Manuscrit reçu
le 31. III 1936.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

TEORETISCHE MECHANIK

ÜBER STATISCH UNBESTIMMTE SYSTEME. DAS ÜBERBESTIMMTE SYSTEM

Von G. KRUTKOW, korrespondierendem Mitglied der Akademie

§ 1. Es bedeute N die Anzahl der Massenpunkte des Systems, $x_1 \ldots, x_n$ ihre Koordinaten $(3N = n)$, $X_1, \ldots, X_n$ die Komponenten der Kräfte, $\varphi_\alpha(x_1, \ldots, x_n) = 0$ die Bedingungsgleichungen $(\alpha = 1, 2, \ldots, n + r)$—dieselben sollen ein gemeinsames Lösungssystem $x_1^0, \ldots, x_n^0$ besitzen—, $a_{\alpha s}$—den Wert von $\frac{\partial \varphi_\alpha}{\partial x_s}$ für die Gleichgewichtslage. Die Reaktionskräfte genügen den Gleichungen:

$$X_s^0 = \sum_{\alpha=1}^{n+r} \lambda_\alpha a_{\alpha s}, \qquad (s = 1, 2, \ldots, n), \quad (1)$$

wo der Index Null «für die Gleichgewichtslage» bedeutet und die λ_α die «reduzierten Reaktionskräfte» (Lagrangeschen Multiplikatoren) bezeichnen.

Die $n + r$ Grössen λ_α können aus den n Gleichungen (1) nicht eindeutig berechnet werden: das kinematisch überbestimmte System ist statisch unbestimmt.

§ 2. Nun kann man, indem man die üblichen Methoden der technischen Mechanik verallgemeinert, für das hier betrachtete allgemeine System folgendermassen verfahren (1):

a) Man ersetze die Bedingungsgleichungen $\varphi_\alpha(x_1, \ldots, x_n) = 0$ durch die «linearisierten» Gleichungen:

$$q_\alpha^* = \sum_{s=1}^{n} a_{\alpha s}(x_s - x_s^0) = 0 \quad (\alpha = 1, 2, \ldots, n + r); \quad (2)$$

b) Man ersetze die Reaktionskräfte durch Potenialkräfte mit dem Potential (2):

$$\Phi = \frac{1}{2} \sum_{\alpha=1}^{n+r} \varepsilon_\alpha q_\alpha^{*2}. \quad (3)$$

c) Die Gleichungen (1) erhalten jetzt die Gestalt:

$$X_s^0 = \sum_{\alpha=1}^{n+r} \varepsilon_\alpha q_\alpha^* a_{\alpha s} \qquad (s = 1, 2, \ldots, n). \quad (4)$$

d) Zwischen $n+r$ linearen Formen von n Veränderlichen $x_s - x_s^0$ $(s = 1, 2, \ldots, n)$ hat man r lineare homogene Relationen:

$$\sum_{\alpha=1}^{n+r} \gamma_{j\alpha}^* q_\alpha^* = 0 \qquad (j = 1, 2, \ldots, r). \quad (5)$$

e) Aus (4) und (5)—insgesamt $n+r$ Gleichungen—lassen sich im allgemeinen die $n+r$ Grössen $q_1^*, \ldots, q_{n+r}^*$ eindeutig berechnen und daraus die Reaktionskräfte.

f) Setzen wir in (4) alle X_s^0 gleich Null und lassen die Gleichungen (5) weg, so gewinnen wir aus (4) Werte der q_α^*, die den «Montierungsreaktionen»(3) entsprechen.

g) Durch Zusammenfassung der Reaktionskräfrte e) und f) erhalten wir die allgemeine Lösung der Aufgabe. Sie erfordert also die Auflösung (in bezug auf die q_α^*) eines Systems von $n+r$ linearen Gleichungen (4) und (5) und ferner eines Systems von n Gleichungen mit $n+r$ Unbekannten.

§ 3. Wir wollen nun zeigen, dass es zur Lösung der Aufgabe genügt ein System von n Gleichungen mit n Unbekannten und ein solches von r Gleichungen mit r Unbekannten zu lösen, was im Falle wenn $n+r$ nicht klein ist, eine nicht unerhebliche Vereinfachung bedeutet. Zu diesem Zwecke drücken wir aus irgend welchen n Gleichungen des Systems (2), das wir jetzt schreiben

$$\sqrt{\varepsilon_\alpha} \sum_{s=1}^{n} a_{\alpha s} (x_s - x_s^0) = q_\alpha, \quad (6)$$

$(q_\alpha = \sqrt{\varepsilon_\alpha} \cdot q_\alpha^*)$, die $x_s - x_s^0$ durch die $q_1, \ldots, q_n$ aus (es sollen die n ersten Gleichungen $\alpha = 1, 2, \ldots, n$ sein). Das ist möglich, da das System (2), resp. (6), nach Voraussetzung nur durch $x_s - x_s^0 = 0$ für alle s befriedigt wird. Wir erhalten, also, durch Lösung des Systems von n Gleichungen

$$x_s - x_s^0 = \sum_{k=1}^{n} A_{sk} q_k \qquad (s = 1, 2, \ldots, n). \quad (7)$$

Wir betrachten nun die q_α $(\alpha = 1, 2, \ldots, n+r)$ als verallgemeinerte Koordinaten des Systems und berechnen die verallgemeinerten Kräfte:

$$\left.\begin{aligned} Q_k^0 &= \sum_{s=1}^{n} X_s^0 \frac{\partial x_s}{\partial q_k} = \sum_{s=1}^{n} X_s^0 A_{sk} \qquad (k = 1, 2, \ldots, n), \\ Q_{n+j}^0 &= 0 \qquad\qquad\qquad\qquad (j = 1, 2, \ldots, r). \end{aligned}\right\} \quad (8)$$

Die Potentialkräfte, welche die Reaktionskräfte ersetzen, sind

$$-\frac{\partial \Phi}{\partial q_\alpha} = -\frac{\partial}{\partial q_\alpha}\frac{1}{2}\sum_{\alpha=1}^{n+r} q_\alpha^2 = -q_\alpha. \tag{9}$$

Die Relationen (5) betrachten wir als r Bedingungsgleichungen für die $n+r$ Grössen q_α. Die Gleichgewichtsbedingungen lauten nun:

$$\left.\begin{aligned} Q_k^0 &= q_k + \sum_{j=1}^{r} \Lambda_j \nu_{jk} \qquad (k = 1, 2, \ldots, n), \\ 0 &= q_{n+i} + \sum_{j=1}^{r} \Lambda_j \nu_{j,\,n+i} \qquad (i = 1, 2, \ldots, r), \end{aligned}\right\} \tag{10}$$

dazu kommen die «Bedingungsgleichungen» (5) $(\nu_{j\alpha} = \nu_{j\alpha}^*/\sqrt{\varepsilon_\alpha})$

$$\sum_{\alpha=1}^{n+r} \nu_{j\alpha} q_\alpha = 0 \qquad (j = 1, 2, \ldots, r). \tag{11}$$

Durch Multiplikation mit $\nu_{h\alpha}$ $(h = 1, 2, \ldots, r)$ und Addition gewinnen wir aus (10), wenn wir (11) berücksichtigen, folgendes System von r Gleichungen zur Bestimmung der Λ_j:

$$\sum_{k=1}^{n} \nu_{hk} Q_k^0 = \sum_{\alpha=1}^{n+r}\sum_{j=1}^{r} \Lambda_j \nu_{h\alpha}\nu_{j\alpha} = \sum_{j=1}^{r} \Lambda_j N_{jh} \qquad (h = 1, 2, \ldots, r), \tag{12}$$

dessen Determinante nicht verschwindet [als Gramsche Determinante der r «Vektoren» $(\nu_{j1}, \nu_{j2}, \ldots, \nu_{j,\,n+r})$ $(j = 1, 2, \ldots, r)$].

Um die «Montierungsreaktionen» zu berechnen braucht man nur die Q_k^0 gleich Null zu setzen und den Λ_j willkürliche Werte zuzuschreiben, also

$$q_\alpha = -\sum_{j=1}^{r} \Lambda_j \nu_{j\alpha} \qquad (\alpha = 1, 2, \ldots, n+r). \tag{13}$$

Um dies zu zeigen nehmen wir etwa $\Lambda_1 \neq 0$, $\Lambda_2 = \ldots = \Lambda_r = 0$, also $q_\alpha = -\Lambda_1 \nu_{1\alpha}$. Setzen wir das in (4) ein, so erhalten wir, indem wir alle X_s^0 gleich Null nehmen:

$$\sum_{\alpha=1}^{n+r} \nu_{1\alpha}\sqrt{\varepsilon_\alpha}\, a_{\alpha s} = \sum_{\alpha=1}^{n+r} \nu_{1\alpha}^* a_{\alpha s} = 0 \tag{14}$$

$\left(q_\alpha^* = \frac{q_\alpha}{\sqrt{\varepsilon_\alpha}},\ \nu_{1\alpha} = \frac{\nu_{1\alpha}^*}{\sqrt{\varepsilon_\alpha}}\right)$. Nun ist

$$\sum_{\alpha=1}^{n+r} \overset{*}{\nu}_{i\alpha} \overset{*}{q}_{\alpha} = \sum_{\alpha=1}^{n+r} \sum_{k=1}^{n} \overset{*}{\nu}_{i\alpha} a_{\alpha k} (x_k - x_k^0) = 0 \qquad (15)$$

für alle Werte der $x_k - x_k^0$, woraus (14) folgt.

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Eingegangen
d. 17. IV. 1936.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

ASTROPHYSIQUE

SUR LA PHOTOMÉTRIE ABSOLUE DE LA COURONNE SOLAIRE

Par B. FESENKOV (FESSENKOFF), de l'Académie

On considère à l'heure actuelle que la couronne intérieure du Soleil est due aux électrons libres et la couronne extérieure, par contre, de préférence à la matière météorique. M. Grotrian conclut de son examen des observations de Ludendorff que la première composante présente une décroissance très rapide de la luminosité avec la distance angulaire du Soleil. La composante météorique est caractérisée par la présence de raies d'absorption toutes semblables à celles du spectre solaire, mais à contraste moins prononcé. Tout ce que nous savons sur la répartition de la matière météorique conduit à penser que la couronne extérieure doit être plus ou moins homogène, abstraction faite des jets coronaux. Malheuresement, les résultats de Grotrian à ce sujet ne sont que qualitatifs, puisque, à sa propre estimation dès une distance de 20′—25′ du bord du Soleil l'atmosphère terrestre fournit pendant l'eclipse totale 60% au moins de la luminosité observée de la couronne. Or, on n'a jamais tenu compte de cette influence atmosphérique d'une façon complète et ainsi la photométrie absolue de la couronne n'était pas possible jusqu'à présent.

La luminosité du ciel pendant l'éclipse dépend de la diffusion de la lumière coronale s'effectuant de façon absolument semblable à la diffusion de la lumière du Soleil présentant la même composition spectrale. Donc, pour l'obtenir il suffit de déterminer l'intensité lumineuse du ciel diurne par rapport au Soleil et ensuite de la réduire dans la proportion de l'éclat intégral de la couronne. Ce n'est qu'à des distances angulaires au-dessous de 30′ du bord solaire qu'il est nécessaire de tenir compte de la différence entre les formes coronales et la figure sphérique du Soleil. Nous supposons dans ce cas que chaque élément $d\delta$ d'un astre cosmique produit un éclairement de l'atmosphère à la distance r égal à

$$dj = (a + be^{-kr})\, i d\sigma,$$

où i est l'intensité intrinsèque de l'élément considéré et a, b, k les paramètres caractérisant l'état de l'atmosphère ainsi que la diffusion de la lumière dans l'optique employée par l'observateur.

D'autre part, l'éclairement du ciel pendant l'éclipse est dû aux rayons du Soleil, provenant des régions non totalement éclipsées, qui pénètrent dans la zône de la totalité. C'est pour cela que le ciel paraît fortement éclairé

près de l'horizon. Cet éclairement observé dans l'azimut A peut être représenté à un facteur près par l'expression générale:

$$j_{A,h} \sim f(\vartheta) \int_{r_\vartheta}^{\infty} L_{A,r}\, p^{\frac{\rho}{\rho_0}\sec\zeta} \sec\zeta\, e^{-k\frac{h}{h_0}} e^{-\int_0^r k\,dr}\, dr, \qquad (1)$$

où $L_{A,h}$ est la quantité de lumière du Soleil partiellement éclipsé à la distance r de l'observateur; $f(\vartheta)$—l'indicatrice de la diffusion atmosphérique qui dépend uniquement de l'angle ϑ entre les rayons incident et diffusé; p—le coefficient de transparence atmosphérique; k—le coefficient d'absorption qui est pris égal à

$$k = -m\frac{\log p}{h_0 \log e} e^{-m\frac{h}{h_0}},$$

h_0—la hauteur de l'atmosphère homogène, ρ—la densité de l'aire atmosphérique; ζ—la distance zénithale du Soleil.

Remarquons que l'indicatrice de diffusion peut être facilement déterminée d'après des observations sur l'éclat du ciel diurne à différentes distances angulaires du Soleil. Les valeurs de $f(\vartheta)$ ainsi obtenues par moi en 1933 et 1934 à l'observatoire Astrophysique de Kutchino peuvent être représentées approximativement par la formule:

$$f(\vartheta) = \mu(1 + \alpha\cos^n\vartheta),$$

où $\alpha = 6.2$ pour $90° \geqslant \vartheta \geqslant 10°$
$\alpha = 0$ » $180° \geqslant \vartheta \geqslant 90°$
$n = 3.6$

La solution du problème toutefois ne peut être obtenue d'une façon bien déterminée tant que le pouvoir diffusant de l'atmosphère aux diverses hauteurs reste inconnu. Pour tenir compte de la diffusion produite par la poussière atmosphérique nous avons adopté d'après les observations de Bemporad $m=4$, c'est-à-dire la hauteur homogène de l'atmosphère diffusante 4 fois moindre que celle de l'atmosphère gazeuse. Pour fixer les idées, supposons que l'observateur se trouve au centre de la région éclipsée près de Krasnoiarsk (Siberie), où le moment de la totalité coïncide avec le midi moyen local (éclipse de 1936). Nous avons calculé suivant la formule (1) l'eclat relatif du ciel près de l'horizon en tenant compte approximativement de la réfraction des rayons diffusés. Les résultats sont donnés dans la table 1 pour des altitudes de l'observateur s'échelonnant de 0 km à 16 km au dessus du niveau du sol.

Table 1

A \ h	0 km	0.4 km	0.8 km	1.6 km	3.0 km	5.0 km	8.0 km	12.0 km	16.0 km
180°	0.489	0.426	0.462	0.521	0.667	0.796	0.852	0.893	0.931
150°	0.589	0.538	0.571	0.608	0.713	0.814	0.907	0.944	0.938
120°	0.754	0.715	0.740	0.765	0.838	0.885	0.970	0.971	0.973
90°	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
60°	0.668	0.615	0.654	0.691	0.816	0.912	1.014	1.002	1.019
30°	0.530	0.442	0.479	0.554	0.772	0.966	1.121	1.109	1.145
0°	0.443	0.373	0.394	0.476	0.719	0.927	1.165	1.192	1.203

On peut également calculer la variation de la luminosité J du ciel au-dessus de l'observateur pendant l'éclipse avec la hauteur. On a:

h	0 km	2 km	4 km	6 km	8 km
J	1.000	0.980	0.873	0.657	0.417

On en voit qu'un observateur effectuant une ascension en avion n'aura pas grand avantage en comparaison avec celui qui demeure sur le sol en ce que concerne l'éclairement général du fond du ciel. Par contre, le halo autour du Soleil venant se superposer à la couronne extérieure doit être considérablement diminué sinon disparaître complètement. Supposons maintenant que chaque élément de l'horizon éclairé puisse être assimilé à une source cosmique placée en dehors de l'atmosphère terrestre. Cela est parfaitement admissible, vu que l'épaisseur diffusante de l'atmosphère est bien petite par comparaison avec la région embrassée par l'ombre lunaire. On peut montrer alors, que l'éclat du ciel en le point (z', A') est donné par l'expression suivante:

$$j_{z',A'} \sim \frac{\int\int f(\vartheta) \frac{p^{m(90°)} - p^{m(z')}}{m(z') - m(90°)} m(z') \mu_h j_{A,h} \, dA \, dh}{\int\int \mu_h j_{A,h} \, dA \, dh},$$

$m(z')$ étant l'épaisseur équivalente de l'atmosphère, $j_{A,h}$—l'éclat de l'horizon dans la direction A et pour l'altitude h, μ_h—le pouvoir diffusant correspondant. On obtient alors les résultats suivants:

Table 2

z' \ A'	180°	150°	120°	90°	60°	30°	0°
0°	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10°	1.05	1.05	1.06	1.06	1.06	1.06	1.06
20°	1.11	1.11	1.11	1.11	1.12	1.12	1.12
30°	1.27	1.27	1.28	1.29	1.29	1.29	1.29
40°	1.53	1.54	1.55	1.57	1.58	1.58	1.58
50°	1.93	1.95	1.97	2.00	2.01	2.02	2.02
60°	2.53	2.55	2.60	2.64	2.66	2.67	2.67
70°	3.39	3.43	3.51	3.57	3.61	3.62	3.62
80°	4.31	4.38	4.50	4.60	4.65	4.67	4.68

Cette table montre que la composante atmosphérique envisagée est presque indépendante de l'azimuth. Elle peut être obtenue facilement des observations à effectuer pendant la totalité en le point du ciel symétrique au Soleil. La méthode développée dans ce qui précède fournit donc la possibilité d'éliminer complètement l'influence atmosphérique et de réaliser la photométrie absolue de la couronne solaire.

Observatoire Astrophysique.
Kutchino.

Manuscrit reçu
le 9. IV. 1936.

Comptes Rendus (Doklady) de l'Académie des Sciences l'URSS
1936. Volume II (XI), № 6 (92)

PHYSICS

SOME REMARKS ON α-AND β-ICE

By N. SELJAKOV

(Communicated by S. I. Wawilow, Member of the Academy, 4. V. 1936)

In the preceding paper «To What Class of Symmetry Does Ordinary Ice belong»(1) we have shown that there are two modifications of ordinary ice: α-and β-ice; α-ice belongs to one of the four classes: D_{6h}, D_6, C_{6v} and D_{3h}, and β-ice belongs to one of the two classes: C_{3i} and C_3.

The measurements of X-ray photographs (Laue method) which we have carried out show that c/a for α ice is equal to 1.62 and c/a for β-ice is equal to 1.33.

The angle between the axes of β-ice is equal to 86°38′, *i. e.* we may consider β-ice as pseudo-cubic.

The value of c/a found by us for the hexagonal form of ice approaches to the corresponding values in the works of Rinne (2) $\left(\frac{c}{a} = 1.6\right)$, Gross (3) $\left(\frac{c}{a} = 1.6\right)$, Dannison (4) $\left(\frac{c}{a} = 1.62\right)$ and Barnes (5) $\left(\frac{c}{a} = 1.634\right)$.

A. John (6) has probably investigated β ice and obtained $\frac{c}{a} = 1.40$.

From the X-ray photographs obtained, we may determine the orientation of α-and β-ice crystals to the normal to the water surface on which ice formation took place. We have found that the angle between the above mentioned normal and the hexagonal axis of symmetry for α-ice was equal from 0°.5 —to 5°, and the angle between the same normal and the axis of symmetry of the third order for β-ice was from 5° to 10°.

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Leningrad

Received
4. V. 1936

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

PHYSIQUE

LA DIFFRACTION DE LA LUMIÈRE PAR DES ULTRA-SONS

Par S. RYTOV

(Présenté par L. I. Mandelstamm, de l'Académie, le 25. IV. 1936)

C. V. Raman et N. S. Nagendra Nath ([1–3]) ont récemment développé une théorie de la diffraction de la lumière par des ultra-sons. Cette théorie permet d'obtenir des formules très compactes pour l'intensité des faisceaux de lumière diffractés et d'expliquer le phénomène de l'apparition d'un grand nombre de spectres de diffraction. Le point de départ de cette théorie est l'hypothèse que l'onde lumineuse traverse la colonne ultra-sonore sans changer d'amplitude et que l'inhomogénéité optique du milieu influe seulement sa phase. En cas d'incidence normale ([1]) cela signifie que les rayons lumineux cheminent parallèlement aux plans d'égale densité dans la colonne ultra-sonore en ligne droite; ayant une même longueur l (fig. 1), ils n'acquièrent une différence de phases que par suite des variations de l'indice de réfraction. Par conséquent, le champ lumineux s'exprime au plan de sortie $z=l$ par la formule:

$$e^{ik(1+\alpha\cos Kx)\,l} \qquad \left(\alpha=\frac{\mu_1}{\mu_0}\right), \tag{1}$$

$K=\frac{2\pi}{\Lambda}$ étant le nombre d'ondes acoustiques, μ_1—l'amplitude des variations de l'indice de réfraction auprès de sa valeur moyenne μ_0, et $k=\frac{2\pi\mu_0}{\lambda_0}$ — nombre d'ondes lumineuses (λ_0 est la longueur d'onde dans le vide).

On déduit alors par le calcul de l'intégrale de diffraction pour le «réseau» (1) que l'amplitude des oscillations lumineuses dans un spectre d'ordre n est proportionelle à $J_n(\alpha\ kl)$, J_n étant la fonction de Bessel d'ordre n.

Se basant sur l'hypothèse exposée, les auteurs ont généralisé cette théorie pour le cas d'une incidence oblique de l'onde primaire ([2]) et l'ont ensuite étendue aux cas d'ondes ultra-sonores stationnaires et mouvantes ([3])*. La théorie dans cette forme se trouve en accord qualitatif avec une série de résultats expérimentaux ([4–7]). Néanmoins, son insuffisance apparaît du fait qu'on en déduit que l'intensité des spectres ne dépend pas de la longueur de l'onde ultra-sonore Λ, en contradiction avec les données expérimentales. Ces dernières, ainsi que la théorie rigoureuse de L. Brillouin ([8]) et P. Debye ([9]),

* Cette extension de la théorie a été faite en partie par E. Hiedemann et E. Schreuer ([11]).

montrent que cette dépendance a lieu, et cela d'une façon d'autant plus marquée que Λ est plus petite ([10]).

Cette circonstance, comme aussi le sens même de l'hypothèse fondamentale de cette théorie (propagation des rayons en ligne droite), montrent que ces résultats ne peuvent être appliqués qu'au cas d'ondes ultra-sonores grandes, ou bien de colonnes ultra-sonores minces. Cette dernière conclusion est confirmée par la vérification expérimentale effectuée par R. Lucas ([12]) et mentionnée par les auteurs eux-mêmes ([13]). On peut évaluer quantitativement les limites d'application de cette théorie par plusieurs moyens, mais de toutes façons le résultat est identique: la théorie de Raman et Nagendra Nath n'est applicable que si

$$\Lambda^2 \gg l\lambda_0.$$

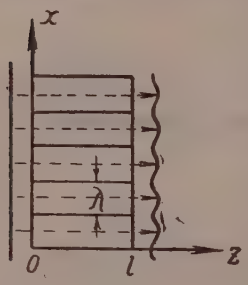

Fig. 1

En adoptant $l = 2$ cm et $\lambda_0 = 4000$A, on a $\Lambda \ll 0.01$ cm, condition qui n'est ordinairement pas remlie de façon suffisante.

L. Brillouin a posé le problème d'une façon qui est sans contredit la plus adéquate et conforme aux conditions réelles. En abordant le problème de même façon, j'ai tâché d'en trouver une solution approximée, plus exacte que la solution de Raman et Nagendra Nath, et se prêtant mieux aux calculs quantitatifs que la solution rigoureuse de L. Brillouin.

L. Brillouin a indiqué ([8]) que le problème de la détermination du champ lumineux au plan $z = l$ se réduit avec un haut degré d'approximation (en négligeant la réflexion de la lumière, les effets minimes de polarisation et en considérant α comme ne dépendant pas explicitement des temps) à la solution de l'équation scalaire:

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2} - \frac{\mu_0^2 (1 + \alpha \cos Kx)^2}{c^2} \frac{\partial^2 E}{\partial t^2} = 0, \qquad (2)$$

les conditions initiales étant données au plan $z = 0$ par l'onde incidente. Si cette dernière est une onde plane d'incidence normale, ces conditions sont:

1) l'absence d'ondes réfléchies, et

2) la condition $E_{z=0} = e^{i\omega t}$.

Naturellement, on peut déterminer avec une grande précision les fréquences des faisceaux lumineux diffractés en posant de nouveau que α dépend du temps dans le résultat final (de façon différente dans les cas d'ondes ultra-sonores stationnaires et mouvantes).

Posant $E = E_0(x, z)\, e^{i\omega t}$, effectuant le changement de variables: $\zeta = kz$, $\xi = Kx$, et mettant:

$$\frac{K^2}{k^2} = \frac{4\pi^2 c^2}{\Lambda^2 \omega^2 \mu_0^2} = \beta, \qquad (3)$$

on trouve la forme:

$$\frac{\partial^2 E_0}{\partial \zeta^2} + \beta \frac{\partial^2 E_0}{\partial \xi^2} + (1 + \alpha \cos \xi)^2 E_0 \qquad (4)$$

pour l'équation (2).

Si l'on pose $\beta = 0$ ($\Lambda = \infty$), on obtient la solution:

$$E_0 = e^{-i(1+\alpha \cos \xi)\zeta}, \qquad (5)$$

qui n'est autre chose que l'expression (1), point de départ de la théorie de Raman et Nagendra Nath. Il est intéressant de noter que lorsque l'on pose

$\beta = 0$, une dérivée de second ordre disparaît dans l'équation de Mathieu, à laquelle se réduit l'équation (4) et dont L. Brillouin recherche une solution. On peut prévoir, par conséquent, que les solutions seront essentiellement différentes dans le cas limite où $\beta=0$ et dans le cas où β n'est pas nulle, même si elle est petite.

Je pose:

$$E_0 = e^{a(\zeta,\,\xi)+i\Phi(\zeta,\,\xi)} \qquad (6)$$

et je développe en série de puissances de $\alpha = \frac{\mu_1}{\mu_0}$ le logarithme de l'amplitude a et la phase Φ:

$$\left.\begin{aligned} a &= a_0 + \alpha a_1 + \ldots, \\ \Phi &= \Phi_0 + \alpha\Phi_1 + \ldots \end{aligned}\right\} \qquad (7)$$

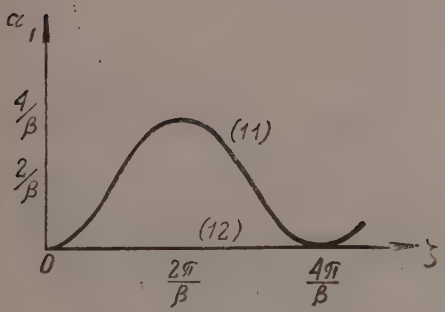

Fig. 2

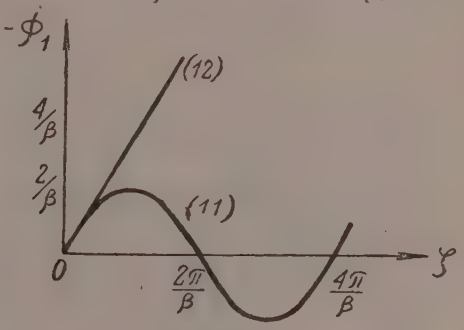

Fig. 3

Lorsque $\alpha=0$ le milieu est homogène et la solution doit avoir la forme: $E_0 = e^{-i\zeta}$. Par conséquent:

$$a_0 = 0, \quad \Phi_0 = -\zeta. \qquad (8)$$

Les conditions initiales pour a_1 et Φ_1 sont donc:

$$\left.\begin{aligned} \zeta = 0, \quad & a_1 = 0, \quad \Phi_1 = 0, \\ & \frac{\partial a_1}{\partial\zeta} = 0, \quad \frac{\partial\Phi_1}{\partial\zeta} = 0. \end{aligned}\right\} \qquad (9)$$

Introduisant (6)—(8) dans l'équation (4), on obtient les équations de première approximation:

$$\left.\begin{aligned} 2\frac{\partial\Phi_1}{\partial\zeta} + \frac{\partial^2 a_1}{\partial\zeta^2} + \beta\frac{\partial^2 a_1}{\partial\xi^2} + 2\cos\xi = 0, \\ -2\frac{\partial a_1}{\partial\zeta} + \frac{\partial^2\Phi_1}{\partial\zeta^2} + \beta\frac{\partial^2\Phi_1}{\partial\xi^2} = 0. \end{aligned}\right\} \qquad (10)$$

La solution de ces équations satifaisant aux conditions initiales (9) est:

$$\Phi_1 = \frac{\cos\xi}{\beta}\left[\left(\frac{1}{\sqrt{1-\beta}} - 1\right)\sin(1+\sqrt{1-\beta})\zeta - \left(\frac{1}{\sqrt{1-\beta}} + 1\right)\sin(1-\sqrt{1-\beta})\zeta\right],$$

$$a_1 = \frac{\cos\xi}{\beta}\left[2 + \left(\frac{1}{\sqrt{1-\beta}} - 1\right)\cos(1+\sqrt{1-\beta})\zeta - \left(\frac{1}{\sqrt{1-\beta}} + 1\right)\cos(1-\sqrt{1-\beta})\zeta\right]:$$

Retranchant les termes qui ne contiennent pas β dans le dénominateur ($\beta \cong 10^{-5}$), on a:

$$\left.\begin{aligned} \Phi_1 &= -\frac{2}{\beta}\sin\frac{\beta\zeta}{2}\cos\xi; \\ a_1 &= \frac{2}{\beta}\left(1-\cos\frac{\beta\zeta}{2}\right)\cos\xi. \end{aligned}\right\} \qquad (11)$$

Si l'on pose $\beta=0$, on obtient le résultat de Raman:

$$\left.\begin{aligned} \Phi_1 &= -\zeta\cos\xi, \\ a_1 &= 0. \end{aligned}\right\} \qquad (12)$$

Les figures 2, et 3 permettent de comparer les formules (11) et (12). Le résultat de Raman est applicable si $\zeta < \frac{\pi}{33}$, c'est-à-dire, si

$$\Lambda^2 > \frac{6}{\mu_0} l\lambda_0 \cong 4l\lambda_0.$$

Notons qu'on passe des formules exactes pour Φ_1 et a_1 aux formules approximées (11) en supposant que $\beta^2\zeta \ll \frac{\pi}{2}$. En adoptant $\Lambda = 0.01$ cm, on a $l \ll 10^4$ cm, condition qui est évidemment toujours remplie.

Ma formule (11) ne diffère mathématiquement de la formule (12) que par la forme de la fonction de ζ par laquelle on multiplie $\cos\xi$. Il en résulte que le calcul de l'intégrale de diffraction (la variable d'intégration étant ξ) aboutit à un résultat semblable: l'amplitude des oscillations lumineuses dans le spectre d'ordre n est proportionelle à la fonction de Bessel d'ordre n J_n, mais avec un autre argument. Il est facile de démontrer que dans ma théorie l'amplitude dans un spectre d'ordre n est proportionelle à:

$$J_n\left(\frac{4\alpha}{\beta}\sin\frac{\beta\zeta}{4}e^{i\frac{\beta\zeta}{4}}\right).$$

En conclusion, notons qu'on peut facilement vérifier l'exactitude plus grande de mes formules en ce qui concerne la relation qui existe entre les amplitudes et Λ. En effet, si l'onde ultra-sonore est si faible que l'on peut remplacer la fonction de Bessel J_1 par le premier terme de son développement en série, l'amplitude du premier spectre est proportionelle à:

$$\alpha\zeta\frac{\sin\frac{\beta\zeta}{4}}{\frac{\beta\zeta}{4}} = \frac{2\pi\mu_1 l}{\lambda_0}\frac{\sin\frac{\pi\lambda_0 l}{2\mu_0\Lambda^2}}{\frac{\pi\lambda_0 l}{2\mu_0\Lambda^2}},$$

en accord avec les résultats obtenus par L. Brillouin [8] et P. Debye [9] par la méthode des perturbations, au cas d'ultra-sons faibles. Cela permet de prévoir que mes formules peuvent être étendues à d'autres problèmes où se révèle la nature spatiale de la diffraction (comme la réflexion de Bragg au cas d'incidence oblique, voir [10]), problèmes que l'approximation de Raman et Nagendra Nath ne peut embrasser.

Dans mon prochain article je compte estimer les limites de la réduction du problème que l'on effectue en négligeant les puissances de α supérieures à la première, discuter les résultats exposés ci-dessus d'une façon plus détaillée et les appliquer au calcul des intensités des faisceaux diffractés.

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Manuscrit reçu le 25.IV 1936.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

PHYSICS

ON THE ABSORPTION OF NEUTRONS IN Ag AND Cd

By N. DOBROTIN

(Communicated by S. I. Wawilow, Member of the Academy, 1. IV. 1936)

This article is a preliminary communication of the results of experiments on the absorption of neutrons in Ag and Cd*. The object of the work was to ascertain in how far the absorption of neutrons by the elements is affected by the thickness of the paraffin plates placed in the way of the neutrons.

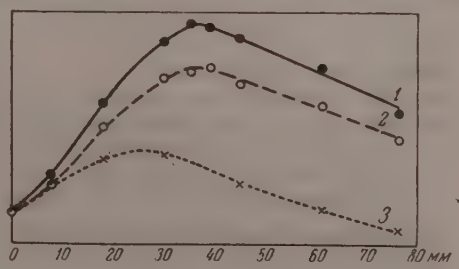

Fig. 1.

The neutrons were obtained by the action exercised on Be by the γ-rays of radon. The ampoule containing the radon was placed along the axis of a cylindrical box containing the Be powder, the diameter of the box being 10 cm, and the thickness of the Be layer 9.5 cm. The neutrons acting on a Geiger-Müller silver counter, induced in the walls of the latter an artificial radioactivity, and this was subsequently measured. The thickness of the walls of the counter was equivalent to a weight of $0.48\ \frac{g}{cm^2}$. The distance between the axis of the counter and the ampoule containing the radon was 15.5 cm. Between the counter and the Be, and close to the counter, were placed the absorbing plates: Ag $\left(0.486\ \frac{g}{cm^2}\right)$ and Cd $\left(0.16\ \frac{g}{cm^2}\right)$. Between the absorber and the Be, and close to the former, were placed rectangular paraffin plates of varying thicknesses, the surface of the plates being $11 \times 6.4\ cm^2$. With a view to absorbing the neutrons scattered in the ambient objects, the apparatus was protected from below and on the sides by layers of borax about 1.5 cm thick. The bombardement of the counter by neutrons lasted for about 10 minutes, wherafter the radon was removed. The counter

* After these experiments had in the main been completed, a letter from I. Kurčatov and G. Ščepkin [1] setting forth analogous results appeared in the press. The results obtained by these two authors agree on the whole with those of the experiments described.

was switched on 42 seconds after the end of the bombardement, the number of throws registered in the space of five minutes being counted.

In Fig. 1 are given the curves illustrating the influence of the thickness of the paraffin layer on the activity induced in the counter.

Curve *1* shows the activity in the absence of an absorber; curve *2* shows the same activity with an Ag absorber, and curve *3* illustrates the same activity with a Cd absorber. Every point of curves *1* and *2* represents 1000—5000 throws; every point of curve 3, 400—1500 throws. These data may also be used to calculate the dependence of absorption on the thickness of the paraffin. This dependence is shown in Fig. 2. On the axis of ordinates has been plotted the ratio of absorption to the activity of the counter in the absence of an absorber; on the axis of abscissae have been plotted the thicknesses of the paraffin. Curve *1* gives the absorption in Ag; curve *2*, the absorption in Cd. As may be seen from the figure, the absorption in Ag remains practically unchanged beginning from paraffin thicknesses of 25—30 mm, whereas the absorption in the Cd increases steadily, its growth slowing down only after great thicknesses of the paraffin have been reached. This points to the fact that the region of an intense absorption of neutrons in Ag begins when the velocities of the neutrons are substantially heigher than is the case with the highly absorbent region of Cd.

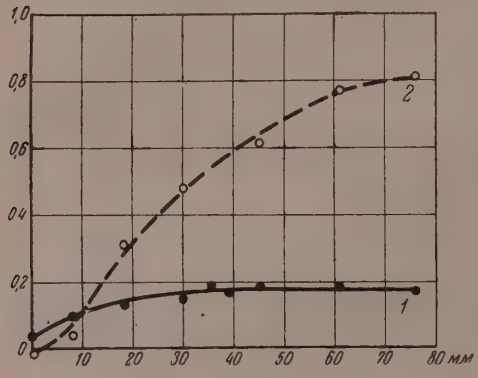

Fig. 2.

According to the data of Fermi and Amaldi ([2]), it appears that, beginning from thicknesses corresponding to a weight of $0.15\ \frac{g}{cm^2}$, Cd absorbes practically the whole of the neutron group which they denote by the letter *C*. As to the absorption of other neutron groups by plates of that thickness, it is negligibly small. Consequently, curve *2* of Fig. 2 shows in how far the ratio of the activity induced in Ag by group *C* to the total activity of Ag due to all the neutron groups is affected by the thickness of the paraffin.

It may thus be seen that so long as the thickness of the paraffin remains small, the number of neutrons of group *C* is relatively small. As the thickness of the paraffin plates is increased, most of the activity of Ag due to neutrons is produced by the neutrons of group *C*.

In conclusion, I wish to record my indebtedness to Prof. D. V. Skobeltsin for the part he has taken in discussing the results of the experiments.

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Received
1. IV. 1936.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

PLANT PHYSIOLOGY

INTENSITY OF GEOTPOPIC REACTION AS A QUANTITATIVE INDEX OF THE CONTENT OF GROWTH-PROMOTING SUBSTANCE

By V. M. KATUNSKIJ

(Communicated by A. A. Richter, Member of the Academy, 28. III. 1936)

Introduction

Went' s method (1) is at the present time the generally used method of measuring the quantity of growth-promoting substance. As is known, this method consists in the following: a black of agar-agar (3%), which has been for some time in contact with the cut surface of the organ under test, is placed on the decapitated coleoptile of the oats, so as not to cover more than half of the cut surface of the organ. After a certain time, the angle of deflection of the coleoptile from the vertical is measured. Went has proved that this deflection measured in angles varies directly as the quantity of growth-promoting substance in the agar-agar block.

The quantity of growth-promoting substance sufficient to cause the coleoptile of the oats to deflect 10° has been termed the «oats unit» (A. E.).

In cases of high concentration, the growth-promoting substance has to be diluted before measurements, for otherwise there develops, as Went has shown, an «inhibitory factor».

Went's method has the following defects:

a) insufficient sensitiveness, allowing an error of $\pm 20\%$;

b) a great variability in reaction owing to exceptional sensitiveness to all kinds of external influence. These defects are most probably due to the fact that the direction of the flow of growth-promoting substance in the decapitated coleoptile varies in dependence on external conditions.

The fact is that the coleoptile-detector in Went's method has a uniform conduction of growth-promoting substance in the basipetal direction, and that the steady flow necessary to cause a deflection is achieved by placing the agar-agar block asymmetrically on the cut surface of the decapitated coleoptile-detector. But under such conditions it is inevitable that, firstly, the growth-promoting substance will spread over the whole surface of the cut coleoptile, thus decreasing the steadiness of its flow; and that, secondly, there will occur considerable variations in the direction of this flow owing to varying asymmetry in the position of the agar-agar block on the coleoptile and the consequent variation in the deflection, etc. It is also possible that in Went's method the coleoptile has to overcome, to a certain degree, its normal negative geotropism, *i. e.* the mechanism controlling the conser-

vation of a strictly vertical position. This circumstance must also lower the sensitiveness of this method.

The present work was undertaken for the purpose of elaborating a method of quantitative determination of the content of growth-promoting substance, which would be free of the above-mentioned defects, *i. e.* would guarantee a sufficient constancy and sensitiveness of the reaction. It would be also desirable to elaborate a method that would not necessitate surgical injury of the experimental plant (decapitation) with the subsequent extraction of growth-promoting substance from it. Such a method would be convenient for studies of the influence of various physical factors on the growth-promoting substance in connection with their action on the growth and development of the plant organism as a whole.

The geotropic response of the seedlings of the oats and the castor-oil plant was chosen as the experimental subject. This choice was due to the fact that the geotropic reaction of seedlings is characterized by great constancy and is a typical growth phenomenon [Kholodny (²), Went and others]. It was therefore natural to assume that the intensity of geotropic reaction measured by the angle of geotropic curvature must vary directly as the quantity of growth-promoting substance and can therefore serve as a quantitative index of its content.

Experimental part

The relation between the intensity of geotropic reaction in decapitated seedlings and the quantity of growth-promoting substances received by them was the first thing to be investigated.

Growth-promoting substance was prepared for the experiments in the usual manner of diffusion extracts in agar-agar blocks (after Went) in different degrees of dilution, namely in the ratio of 1:2:3:4:5.

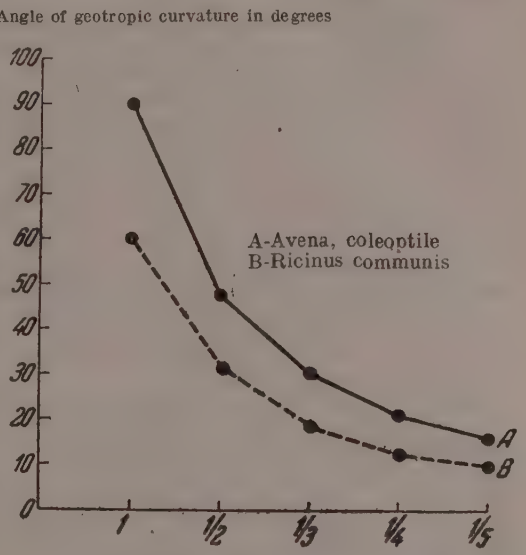
Dilution was also effected after Went's diffusion method. The agar-agar blocks containing different concentrations of growth-promoting substance were concentrically implanted on decapitated coleoptiles of the oats and on decapitated ten-day old seedlings of the castor-oil plant laid in a horizontal position. 2—4 hours later, the angle of deflection of the seedlings from the horizontal was measured. Decapitated seedlings laid horizontally served as controls, and the experiment was considered successful only if their geotropic curvature did not exceed 10°. Simultaneously measurements were carried out in the usual method after Went.

All the experiments were made in the dark, in moist chambers under a constant regime of temperature and moisture. Red light was used setting up the experiment.

The relation obtained between the values of geotropic curvature and the quantity (concentration) of growth-promoting substance is represented graphically in Fig. 1. These curves have a hyperbolic character and show the value of geotropic curvature to vary directly as the concentration of growth-promoting substance, both in coleoptiles of the oats and in seedlings of the castor-oil plant.

This was followed by several series of experiments for the purpose of determining the relative accuracy $\left(\text{variation coefficient equal to } \sqrt{\frac{\Sigma d^2}{n}} \cdot \frac{100}{m} \text{ in } \%\right)$ of mesurements after Went's method and after the one elaborated by us. Coleoptiles as similar as possible were selected, de-

capitated and then had similar blocks of agar-agar with growth-promoting substance implanted on them. Half of these coleoptiles were then left in a vertical position with excentrically implanted blocks and half of them were fixed in a horizontal position with the block placed concentrically. After the tips of the coleoptiles (maize) used as the source of growth-promoting substance were removed from the agar-agar plate (from which the agar-agar blocks were later to be cut out), this plate was kept for 3—4 hours to allow the growth-promoting substance to diffuse uniformly throughout it so as to make sure that the content of growth-promoting substance in the blocks of agar-agar was equal.



The blocks cut out of this plate were regularly (chequer-wise) arranged in two groups corresponding to the two methods of measuring to be compared.

The results of one series of these experiments are given in the following table.

Coleoptyles of Oats

Nos of parallel experiments	Angle of curvature in degrees	
	Obtained after Went's method	As a result of geotropic reaction
1	40	80
2	65	76
3	45	85
4	62	72
5	55	78
6	58	80
7	44	80
8	53	70
9	62	83
10	65	76
Relative accuracy (variation coefficient)	15.81%	5.66%

The decrease of the coefficient of variation and, consequently, of the error of measurements obtained in the case of our method, can be explained

to a considerable degree, by the fact that our method ensures a better uniformity in the direction of the flow of growth-promoting substance in experimental seedlings.

The works of Kholodny, Went and others, have actually established the fact that the geotropic curvature is really associated with the asymmetrical conduction of the growth-promoting substance in the geotropically stimulated organ. Although this asymmetry of conduction is not absolute, *i. e.* although a certain amount of growth-promoting substance flows through all the points of a transverse section of the seedling, it is nevertheless always to be observed owing to some as yet unknown mechanism [perhaps to the «geoelectric polarization» of Brauner, Amlong ([4])]. Still more important is the fact [see Rawitscher ([3])] that, with uniform material (*e. g.* oats coleoptiles and under uniform experimental conditions, equal geotropic reactions are stimulated by similar degrees of asymmetrical distribution of growth-promoting substance in the reacting organs (*e. g.* coleoptiles).

Our method, therefore, unlike that of Went, ensures the necessary stability in the conduction of growth-promoting substance, provided the experimental material is uniform.

In Went's method the conduction varies considerably and is affected to a great extent by the operations of the experimenter involved by this method (implantation of the block, etc.).

On the basis of all the above the following conclusions should be drawn:

1. Intensity of geotropic curvatures of the coleoptiles of oats and the seedlings of the castor-oil plant (measured in degrees of the angle of curvature) varies directly as the quantity of growth-promoting substance and can serve as its quantitative index.

2. The main defect of Went's method is the instability of the conduction of growth-promoting substance, this conduction depending exclusively on the manner the agar-agar block with the growth-promoting substance is implanted, on the state of the cut surface, etc. The amount of the curvature depends in such a case, not only on the amount of growth-promoting substance supplied to the coleoptile, but also on the degree of the stability of its conduction (which in this case varies).

3. The main advantage of our method of quantitative determination of the content of growth-promoting substance, as compared with that of Went, is the stability of the conduction of growth-promoting substance in the geotropically stimulated organs. The angle of curvature in this method depends exclusively on the content of growth-promoting substance.

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Received
28. III. 1936

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

PLANT PHYSIOLOGY

GROWTH-PROMOTING SUBSTANCE AND THE FORMATIVE ACTION OF LIGHT ON VEGETATION IN PLANTS

By V. M. KATUNSKIJ

(Communicated by A. A. Richter, Member of the Academy, 28. III. 1936)

Together with temperature and water regime, light belongs to the most powerful environmental factors having a formative influence on plant organisms. A large number of articles have been devoted to studies of the mechanism of the formative action of light, but the real nature of the relation existing between the action of light and the changes in the form of the plant is hardly yet understood. The present work was undertaken with the object of discovering whether there exists any relation between the formative action of light and its influence on growth-promoting substance. Indirect evidence of the existence of such relation is to be found in the data of Blaauw's (1) and Brauner (2) investigations on photoreaction of growth, wherein they observed photochemical interrelations, in particular the Bunsen-Roscoe law, and in the investigations of Went (3) on phototropism, wherein he succeeded in establishing the destructive action of light on the growthpromoting substance. It seemed that, for the solution of the problem we hadset ourselves, it would be interesting to study the possible changes in the content of growth-promoting substance under the action of illumination of short duration, which, as is well known, produces considerable formative effect.

Experiments were made with etiolated coleoptiles of the oats (a convenient subject for measuring growth-promoting substance and growth) in the following variants: a) continuous darkness, b) 16 hours of illumination per day, and c) d), e) and f) 1, 2, 10, 60 minutes of illumination per day respectively. Electric vapour lamps were the source of light.

Measurements of growth-promoting substance were made after Went and after our method (4).

The results of one of these experiments is represented in the following diagram, where we have plotted along the axis of ordinates a) total growth per 24 hours and b) quantity of growth-promoting substance in terms of its percentage content in fully etiolated coleoptiles (continuous darkness).

The above data show that illumination of short duration causes an almost instantaneous fall in the content of growth-promoting substance to a definite level, while further increase in the duration of illumination can hardly effect a further fall.

Experiments were then carried out with regularly repeated illumination of short duration. All the variants of these experiments were given a similar number of minutes of illumination, namely 24 per 24 hours. The difference between the variants consisted exclusively in these 24 minutes of light being given in the different variants in equal portions but at different intervals of time, namely 1, 2, 4, 6 and 8 hours, so that the duration of each period of illumination in the corresponding experiments were 1, 2, 4, 6 and 8 minutes respectively. The experimental material were coleoptiles of the oats. Measurements of growth-promoting substance were made before each new dose of illumination. The results of measurements of growth-promoting

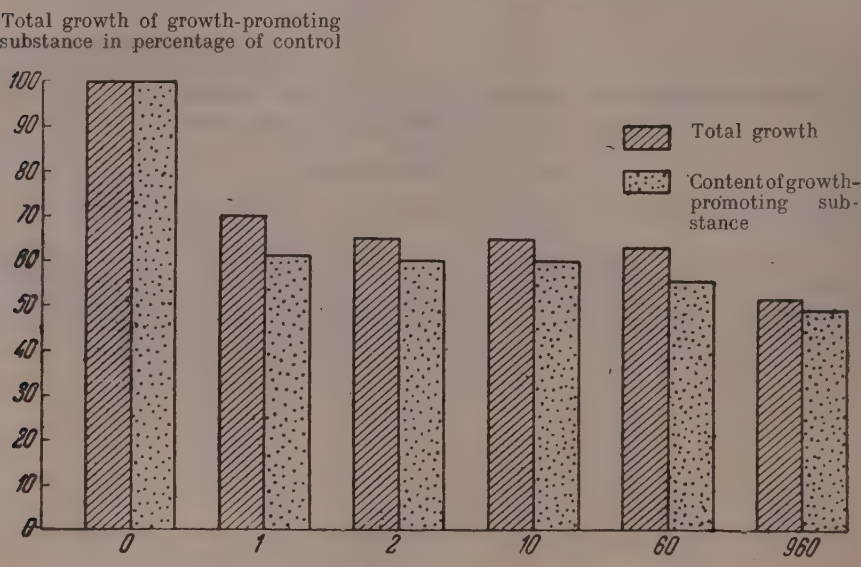

Fig. 1

substance in the variants *A*, *B* and *E*, illuminated every 1, 6, and 8 hours respectively, are represented in the following curves showing the dynamics of the content of growth-promoting substance (in terms of its percentage content in etiolated seedlings). The amount of the total growth during 24 hours in the variants *A*, *B* and *C* was 40, 60 and 67% of maximum growth in continuous darkness respectively.

The above experimental data show that the manner in which the illumination is supplied in respect of time is of immense importance. As may be seen from the above curves, the decisive factor is the time needed for the growth-promoting substance to be renewed, *i. e.* the rate of its physiological regeneration. It is important to note that our data agree with those of Went, Dolk (5) and other writers, who found this time to lie within the limits of from 3 to 6 hours.

The seedlings of the last experiment were subjected, after its completion, to anatomical examination, which revealed that those variants which had shown greatest growth were, at the same time, characterized by the least differentiated conducting system.

Illumination of short duration periodically repeated at different intervals of time was also applied to seedlings (5—7-days old) of the castor-oil plant (*Ricinus communis*). These experiments yielded results which were completely identical with those described above (for the oats). We shall cite only the relative results of the total growth of leaf surface (of the first two leaves) in the *A*, *B* and *C* variants, analogous to those in the experiments with the oats. This growth amounted during 24 hours to 118, 65 and 25% of the growth of the control seedlings cultivated under a normal day (16 hours of illumination) respectively.

All the above warrants the following conclusion: 1) The formative effect of short illumination—elongation of the stalks, development of the leaf sur-

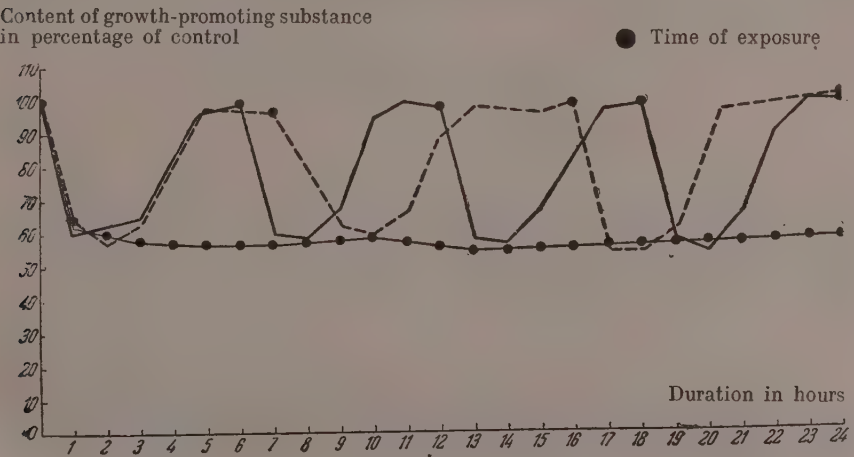

Fig. 2

face area, differentiation of the conducting tissues—depends directly on the decrease in the content of growth-promoting substance caused by it; 2) The formative growth-promoting after-effect of the action of illumination of short duration is limited by the time (rate) of the renewal of growth-promoting substance to the level corresponding to the conditions of growth in the dark or with lack of illumination; 3) Continuous illumination is not necessary for keeping the content of growth-promoting substance in etiolated seedlings at a low level; periodical short (1—5 minutes) illumination, at intervals less than the time necessary for the renewal of growth-promoting substance to the level corresponding to conditions of darkness or insufficient intensity of illumination, is enough for this; 4) So-called etiolated plants differ from those cultivated under normal conditions of illumination in their higher content (overproduction) of growth-promoting substance under conditions of darkness or insufficient illumination; 5) Short periodical illumination may be used as a means of controlling the undesirable phenomena of etiolation, which develop mainly under conditions of cultivation in covered soil (under glass).

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Received
28. III. 1936

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI). № 6 (92)

PLANT PHYSIOLOGY

THE PHYSIOLOGY OF THE INCUBATION PERIOD IN RUST INFECTIONS

By A. I. GREČUŠNIKOV

(Communicated by A. A. Richter, Member of the Academy, 28. III. 1936)

The physiology of the sick plant in the first stages of disease remains a field of phyto-pathophysiology but little investigated. The absence of the external characters of disease render early diagnosis difficult, while existing methods of microscopic observations, not to mention their tediousness, cannot give a clear conception of the physiology of the systems of the higher plants and parasites.

A plant infected by a parasite changes only in the energy of those processes that go on even before the beginning of infection (1).

The investigations published by Schmidt (2) on the diagnosis of early stages of disease of sugar beet infected by rust (*Uromyces betae*) arose great interest. Schmidt, making a microanalysis in fat, found droplike accumulations of lipoid in the cells of an infected leaf staining in sudan and in strong sulphuric acid to a ruby red tint. Normally, the sugar beet does not form fat in its leaves, and the observations of Schmidt indicate the possibility of the appearance of new processes brought about only by infection.

Schmidt's simple method of establishing infection by rust promised wide possibilities of diagnosing the beginning of the development of fungus in plant cells.

In our own work we decided to employ Schmidt's method for determining the infection by rust of a number of cereals, the sunflower, and the sow thistle.

Guided by Schmidt's work, we found fatty inclusions in not a single case. Staining by sudan in the presence of strong sulphuric acid was, indeed, observed, but the stained bodies always manifested a nuclear structure. It may be remarked that the nuclei were noticeably increased in size.

In our opinion, other stained bodies of some sort were taken for globules of fat in Schmidt's investigations—possibly cell nuclei; we know of no facts confirming Schmidt's observations in any other investigations.

Having obtained no results by Schmidt's method, we decided to test the respiration of plants during the incubation period after infection with rust.

Our work was carried on with oats and *Puccinia coronifera* during June—August 1935.

The oats were sown in pots in a room with scattered light and a relatively constant temperature of 18—20° C. Spores of *P. coronifera* were collected from field crops of oats; they showed a high virulence in preliminary cultures; the appearance of pustules of *P. coronifera* on young oats of 3 to 4 blades was always observed in our experiments after 6 days.

Inoculation with the spores was carried on by spraying with a suspension in water by means of an atomizer. We covered the plants with spores thickly enough to obtain a maximum infection judging by pustules. Determination of photosynthesis was carried on by the method of Polovtsev-Richter in a mixture of air and carbon dioxide (CO_2=4.55—4.685 %, O_2=20.76—20.86 %) in the constant light of an electric lamp of 500 candle power at a distance of 25 cm with absorption of the heat rays by a water screen. The exposure lasted 20 minutes. Respiration was determined by the same method, but in an atmosphere of air with an exposure of 1 hour in a dark thermostat.

The experiments began on the second day after infection and were finished after 8 days, when the whole leaf surface was thickly covered with pustules.

The data on photosynthesis are shown in Table 1.

Table 1

Photosynthesis during the Incubation Period

Exposure, 20 minutes; Experimental temperature, 24°C.

Days after infection	Absorption of CO_2 in mm^3 per hour for 1 dm^2 of leaf surface		Percentages	Discharge of oxygen in mm^3 per hour for 1 dm^2 of leaf surface		Percentages	$O_2 : CO_2$	
	Healthy	Infected		Healthy	Infected		Healthy	Infected
4	7 867	11 376	144.6***	5 715	9 209	161.7	0.72	0.81
6	8 932	8 624	96.5	6 110	6 634	108.6	0.68	0.77
7*	6 848	5 336	77.9.	5 015	3 776	73.5	0.73	0.71
8**	9 267	6 320	68.2	8 489	6 982	83.1	0.92	1.11

It is evident from the figures that strong infection raises the photosynthesis during the first days, but from the beginning of the parasite's sporulation (6th day), the photosynthesys drops in energy and continues low thereafter (3).

The results of the investigation of the respiratory processes are presented in Table 2.

While in photosynthesis we had to do with a depression, in the matter of respiration we see the reverse: it remains intensified the whole time; the opening of the rust pustules after 8 days intensifies the respiratory process still more.

The qualitative side of the respiration, which is shown by respiratory coefficients, is interesting.

The respiratory coefficient of the infected leaf drops sharply at first, creating an impression either of incomplete oxidation or of a burning of strongly reduced substances; the onset of spore formation abruptly increases the respiratory coefficient.

It is interesting to note that the rise of the respiratory coefficient is accompanied with appearance of urea in the leaf (qualitative reactions).

* Pustules formed within the tissues with yellow spotting.
** Pustules broke through to the surface of the leaf.
*** Data for healthy plants are taken in all cases as 100.

Table 2

Respiratory coefficients and respiration during incubation period

The control is taken as 100 in all cases. Experimental temeperature, 24°C; Exposure 1 hour.

Days after infection	Respiratory coefficient		Discharge of CO_2 in mm^3 per hour for 1 g raw weight		Percentages***	Absorption of O_2 in mm^3 per hour for 1 g of raw weight		Percentages***
	Control	Infected	Control	Infected		Control	Infected	
2	1.12	1.13	215.00	282.00	131.10	191.50	250.00	130.50
4	0.96	0.65	198.50	203.00	102.30	201.50	313.00	155.30
6	1.03	0.53	153.00	201.50	131.70	149.00	377.00	253.00
7*	1.35	1.16	218.00	263.50	120.90	161.00	228.00	141.60
8**	1.07	1.50	196.00	321.00	163.80	183.00	214.50	117.20

The incubation period ensuing on strong infection with rust is characterized quantitatively and qualitatively by a change in the processes of gas exchange. The photosynthesis after an increase during the first days of infection, falls and remains low even after the appearance of pustules; the respiration is increased, the respiratory coefficient being quite low before the spore formation of the parasite and abruptly increasing after it; intensification coincides with the appearance of urea in the tissues.

No formation of fat in the assimilatory cells following infection (Schmidt's observation) was to be observed.

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Received
28. III. 1936.

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* Pustules formed within the tissues with a yellow spotting.
** Pustules broke through the epidermis to the surface of leaves.
*** Gas exchange of healthy plants taken as 100.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI), № 6 (92)

GEOLOGY

NEW DATA ON THE GEOLOGICAL STRUCTURE OF THE REGION OF THE TAIMYR FOLDING*

By G. D. ALLER

(Communicated by V. A. Obručev, Member of the Academy, 9. III. 1936)

The most ancient rocks outcropping here are metamorphic rocks, considered to be of Proterozoic age. They are represented by the following series: 1) The Waksel series,** consisting of variegated, brecciated highly rolled rocks resembling the augen-gneisses; 2) the series of crystalline schists—garnet-micaceous, zoisite-micaceous, muscovite-biotite and pyroxene schists; 3) mylonites and other cataclastic rocks. The thickness of the whole Proterozoic series is great, attaining approximately 5—6 km. The Proterozoic rocks are most widely distributed in the Islands of Nordenskjold Archipelago on the Coast of Hariton Laptev, in the King Oscar Peninsula, the Byranga Range, in the eastern part of Cheluskin Peninsula and in the eastern part of the islands of the Severnaya Zemlia. All this group of rocks is crumpled, crushed, sheared, faulted and dislocated so as to form compressed and disrupted folds. The axes of the folds form and arch, its convex side facing towards South-East.

The axes of the folds are first directed ENE (King Oscar Peninsula), then 25° NE, in the eastern part of the Cheluskin Peninsula, where the strike of the Pre-Palaeozoic rocks is established on the basis of the Waksel series, which was a key-horison and which could be traced for a distance of 30—35 km (from the mouth of the St. Thaddeus river to the cape of the same denomination). Thus far we do not possess any data indicating the presence of a Pre-Paleozoic phase of folding. By the close of the Proterozoic, the whole region of the Taimyr folding had been highly eroded and represented a peneplain.

The Lower Cambrian epoch is characterized by the submergence of the described region and by the accumulation of thick conglomerate beds containing pebbles, often composed of the Proterozoic metamorphic rocks, by argillaceous and marly slates, chlorite and cericite schists, limestones,

* Under the term of the Taimyr folding we embrace the territory of the Taimyr Peninsula, the King Oscar Peninsula, the Cheluskin Peninsula, the islands of the Nordenskjold Archipelago, Russian Islands, the Severnaya Zemlia, Little Taimyr, Starokadomsky, and Samuel Islands.

** The Waksel series is called after the locality of the first discovery of brecciated rocks at Cape Waksel (eastern shore of Cheluskin Peninsula).

marls and arenaceous shales with scanty remains of fauna. All the Lower Cambrian rocks have to a certain degree undergone dynamo- and pyrometamorphism, as a result of which the argillaceous shales are converted into phyllites, limestones into crystalline limestones, and so on. Nearly all these rocks are highly pyritized and interspersed with quartz and calcite veinlets and large quartz veins. The Lower Cambrian is widely distributed over the Cheluskin Peninsula in its central and northern parts, in the Severnaya Zemlia and in the central part of the Taimyr Peninsula.

The thickness of the whole series equals 3—4 km. The Lower Cambrian sea is succeeded, without any conspicuous breaks, by the Middle Cambrian, characterized by rather quiet conditions during the whole Middle Cambrian epoch. Its deposits are represented by a uniform series of gray and green argillaceous and wacke shales with lenses of sandstone, the whole containing a trilobite fauna. The thickness is rather great. Urvanzev has established that the thickness of the Lower and Middle Cambrian deposits in the Severnaya Semlia attains 7 km. At the beginning of the Upper Cambrian epoch the sea became deeper, an accumulation of sediments of agillaceous facies taking place at its bottom. These sediments are represented by black and green argillaceous slates and arenaceous chales, in places by tuffaceous shales, dyke quartz porphyries of the type of flows. By the close of this epoch, the sea had noticeably shallowed, and marly sandstones of brick red, rarely greenish colours with the fauna of *Obolidae* were deposited. The thickness of the Upper Cambrian attains 2 km. The variation of the Cambrian sea level occurring throughout the whole period seem to have been caused by epeirogenetic movements. The shallowing of the sea which set in in the Upper Cambrian preserved its nature in the Lower Silurian epoch (in its first half). Only in the second half of this epoch a noticeable deepening of the sea was to be observed, as witnessed by a change in the nature of sediments. By the close of the epoch, marls of red, lilac, green and other colours, with the subordinated sandstones of Lower Silurian, had been replaced by sediments of deep sea facies—limestones (marly) and dolomites.

The Lower Silurian sea had a rich fauna. Among the fossil remains, *Trilobites*, *Orthoceratites*, *Crinoids* and other forms occur in great quantities. The fauna and the lithological nature of rocks of this age point indicate that the sea was deep. The thickness of the Lower Silurian attains 2 km. The deepening of the sea which took place in the Lower Silurian epoch had attained its maximum by the close of the Upper Silurian, as witnessed by the deep-water facies: gray massive and bedded limestones, often bituminous, with a fauna of corals (*Halysites*, *Favosites* and others) and *Stromatoporoides*, marly green limestones with *Orthoceratites* and *Brachiopodes*. The upper part of this series consists of marl and sandstone layers, the whole attaining thickness of 1 km. This vast series of limestones could be accumulated only in quiet waters of a deep sea. In the second half of the Upper Silurian the variation of the sea level caused a frequent change of the lithological composition of rocks,—in ascending order: alternation of limestones and dolomites passing into marls with interbedded sandstones (the latter show the lagoonal nature of sediments). The series is crowned by marly and reddish brown sandstones, red and lilac marls with gypsum lenses. The thickness of this series overlying the limestones exceeds 1 km.

At the close of the Upper Silurian, vigorous organic movements (a phase of the Caledonian Cycle) took place. Series of rocks are folded into steep, compressed folds, often reversed to the NW, their axes forming arches with their convex side facing SE. Thus, on the Taimyr Peninsula the strike of

rocks is 50°—75° ENE (Taimyrka river), in the Cheluskin Peninsula it is 25—35° NE, in the Severnaya Zemlia, from 20° NE to 340° NW.

These orogenic movements were accompanied by volcanic phenomena. This first phase of volcanic activity is characterized by powerful granite-gneiss intrusions developed in the eastern part of the Cheluskin Peninsula (Mount Urvanzev and Mount Fersman). The continental uplift took place immediately after, and probably even simultaneously with, the phase of folding. By the close of the Silurian, the sea had receded from the territory under discussion. Thus, the whole region was land during the whole Devonian and probably during a considerable part of the Lower Carboniferous; under such conditions, erosion of the pre-Paleozoic and Lower Paleozoic rocks took place. No continental deposits of Devonian and Carboniferous on the described territory are known.

The phase of slow subsidence of the platform ensued, the process having apparently started in the Turnais, to judge by the analogy with other regions (the Tungus basin). This involved a transgression of the Carbono-Permian sea. The sea overflooded a vast territory: the whole region of the Taimyr folding, the Anabar massif, the Norilsk and Tungus basins. This sea was shallow and, by the close of the Permian period, was transformed into a system of lakes and lagoons separated by swamped strips of land covered with a rich vegetation which served as material for the formation of coal. The lithological composition of Carbono-Permian rocks is rather uniform. They are represented by argillaceous and arkoze sandstones and effusive rocks of the diabase and palaeobasalt type.

Sediments of this age occur in the Severnaya Zemlia (island of the October Revolution), in the southern part of the Taimyr Peninsula, in the region of the Khatanga Gulf, probably to the east of Taimyr lake in the Pronchishchev Bay.

The thickness of this so-called Tungus coal-bearing series is after Urvanzev's data approximately 100—150 m in the Severnaya Zemlia, increasing to the south of Taimyr. The Carbon-Permian deposits are widely distributed to the South and to the South-West of the discribed region, where their thickness differs considerably from that mentioned above.

On the boundary of the Palaeosoic and the Mesozoic new orogenic movements are to be observed (a phase of the Variscian Cycle). As a result of these movements, the whole complex of the Upper Palaeozoic rocks was uplifted in gentle folds whose axes have a strike of 91° SE, *i. e.* close to the latitudinal or almost latitudinal. The Proterozoic and the Lower Palaeozoic rocks were repeatedly disturbed during this orogenic phase, which resulted in a secondary folding, as well as in tangential disruptions and overthrusts, great ones, of the overfold type (charriage), and small ones, of the imbrication type. With this phase of folding are connected the manifestations of vigorous volcanic action. To this period belong powerful intrusions of granites, gabbro-diabases and also effusions of palaeobasalts of the Boris Vilkitsky islands and of Hariton Laptev Cape (the eastern coast of Cheluskin Peninsula). At the beginning of the Mesozoic era a considerable part of North Siberia was land

From the beginning of the Triassic began a continental phase which lasted till the beginning of the Jurassic. Then a subsidence of separate areas took place, followed by the transgression of the Jurassic (Liassic) sea. The presence of the Jurassic deposits is established mostly in the south-eastern part of the region described (the region of the Khatanga Gulf) and in the north-eastern part of the Cheluskin Peninsula (the Lasinius Cape). The

Jurassic deposits are represented by 1) coarse-grained micaceous and calcareous sandstones containing a rich fauna (Liassic): *Tancrediae*, *Pseudomonotes*, *Cyprinae*, and others; 2) brownish-grey calcareous sandstones with a fauna of the Upper Oxford age: *Cardioceras alternans*, *Aucella Bronni*, *A. lata*, *Oxytoma*, and others; 3) coarse-grained micaceous sandstones with a fauna of the Lower Walangine: *Aucella terebratula*, *A. keyserlingi*, *Rhinchonella*, *Belemnites*, *Oxytomae*, *Ditripa* (?), and others; 4) areno-argillaceous continental deposits with plant impressions and thin layers of brown coal.

The subsidence of the continent gave rise to the development of lagoon and lakes. During the Jurassic and the Lower Cretaceous the accumulation of shallow water deposits took place, with which the presence of oil in the Khatanga Gulf seems to be connected. In the beginning of the Tertiary, large radial disruptions occurred in the described area, as a result of which the Mesozoic deposits were broken by a number of faults of various directions. In the region of the Cape Nordwick salt dome, weathered oil is met in fault fissures. In places, the Mesozoic deposits, owing to the radial displacement, got slightly inclined. The disruption was followed by the uplift of separate blocks (involving horsts and grabens). Some grabens seem to have been overflooded by the Tertiary sea. Marine Tertiary sediments are met with on the Cheluskin Peninsula in the region of the Cape Amundsen, Lasinius and Wisting, where they are represented by 1) areno-argillaceous feebly laminated shales, 2) thin-bedded shaly clays with sandstone interlayers, 3) sandy clays. Their thickness amounts to 25—30 m. Besides marine Tertiary deposits, continental ones occur in the central part of the Taimyr Peninsula, along the Boganida and Taimyrka Rivers. At the end of the Tertiary and in the beginning of the Quaternary, the described area underwent a continental phase and seems to have been considerably uplifted. Severnaya Zemlia, Little Taimyr, Starokadomsky and Russian Islands, the Nordenskjold Archipelago, etc. were a continuation of the Taimyr and the Cheluskin Peninsulae.

The beginning of the Quaternary is characterized by a gigantic glaciation of the whole north of Siberia, including the region of the Taimyr folding. The glaciets were of a continental nature. The Severnaya Zemlia archipelago and the central part of the Taimyr Peninsula were the centre of the glaciation. During the long glacial period enormous ice masses were accumulated, owing to which the isostatic equilibrium was disturbed, this being partly the cause of the subsidence of the region. Epeirogenetic movements of the earth's crust were the main cause of the transgression of the Arctic Sea.

Traces of a boreal transgression are to be observed at a height of 119 m above the present level of the Kara sea (the Cheluskin Peninsula). The sediments of the Boreal Sea are represented by sands, sandy and muddy clays and gravels with a fauna of *Astarte borealis*, *Pecten islandicus*, *Mya truncata*, *Saxicava arctica*, *Balanus*, and other forms.

The Boreal Sea spread to the south as far as 67.5° N. Then followed a new phase of uplift of the whole region, and a second glaciation not so strong as the first one. Relicts of the second glaciation have been preserved in the Severnaya Zemlia in the form of dying island glaciers. The second glaciation was followed by the subsidence of certain areas, involving a second transgression of the sea. Traces of the second transgression are to be observed on the Cheluskin Peninsula at a height of 25—30 m above the present level of the Kara sea and on the Taimyr Peninsula. At present, the region of the

Taimyr folding is undergoing a phase of uplifting expressed in a negative shifting of the shore-line.

Received
9. III. 1936.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1936. Volume II (XI). № 6 (92)

ZOOLOGY

THE INFLUENCE OF THE ACTIVE REACTION OF HYDROGEN IONS AND OF SALINITY ON THE EGGS OF *ENGRAULIS ENCRASICHOLUS* L.

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(Communicated by S. A. Zernov, Member of the Academy, 2. III. 1936)

The purpose of the present expedition was a comparative study of the action of salinity and of the active reaction of hydrogen ions on the eggs of *Engraulis encrasicholus*, the anchovy.

The Greeks call this fish khamsa and this name is used on the shores of the Black and Azof Seas.

Engraulis encrasicholus is a pelagic fish, living within the limits of the USSR, in the Azof-Black Seas basin, where there are two subspecies, the Black Sea *Engraulis encrasicholus ponticus*, living in the Black Sea, and another, *Engraulis encrasicholus maeoticus* migrating with the onset of the summer warming of the waters into the Azof Sea, where it spawns and fattens. In the autumn, with the lowering of the temperature of the waters shoals of anchovies go out through the Kerch Straits into the Black Sea where they winter.

The spawning of both subspecies of *Engraulis encrasicholus* begins about the middle of June and lasts till the middle of August. The Azof expedition found separate eggs until September in tests of the plankton. For spawning, these fish seek out places that are protected from waves. The spawning itself usually takes place before dawn, when the waves have died down.

The process of spawning was observed once in the Novorossiisk district. A shoal of fishes passed near Cape Khako, not far from the entrance into Novorossiisk Bay, pursued by dolphins. After the passage of the shoal, the Zepelin nets brought up absolutely fresh roe, with no signs of division.

In the Azof Sea the spawning of *Engraulis* was observed behind Enikalsk lighthouse, where there are small inlets; here also we obtained roe that had just been emitted. The egg of *Engraulis* is perfectly transparent and its stage of development may be roughly determined under the magnifying glass with a magnification of $\times$ 12.

The development of the egg takes place normally in 24 hours at a temperature of 18° C. The fully formed young fish leaves the envelope of the egg and begins its independent life, nourished for the first 48 hours by the supplies of nourishment stored in the yolk sac, after which it begins external feeding.

The eggs of *Engraulis* are usually suspended at a depth of from ½ to 1 m. In collecting them, mechanical injury and rise of temperature, which is fatal to the vitality of eggs and larvae, must be avoided. The catching of eggs in the sea is best carried out by means of a Zepelin net with a removable cuplet. The eggs of the Azof subspecies was taken for experiments from beyond the Enikalsk lighthouse. The eggs of the Black Sea subspecies was taken near Novorossiisk Bay. The eggs of various catches was found in different stages of development, for the most part in that of 64, 32 and 16 blastoemeres, the latter stage being the earliest.

M e t h o d s. All experiments in investigation of the effect of salinity and of the active reaction were carried out in sea water from the corresponding water basin.

Hypotonic solutions were prepared by diluting sea water up to the concentration needed. Inasmuch as bicarbonates break down under evaporation and $CaCO_3$ precipitates, thereby destroying the equilibrium of the salts, a current of CO_2 washed in soda was injected for 3—4 hours in order to restore $CaCO_3$.

In experiments on the influence of pH, sea water, acidulated by the addition of 1/10 *n* HCl to pH 3.5—4 (to complete decomposition) was taken.

In order to remove CO_2 a current of air was passed through the water, which led to a degree of neutralization, and then, by the addition of 0.02 *n* NaOH the solution was brought to the needed concentration of hydrogen ions.

The concentration of hydrogen ions was determined colorimetrically by Clark's method. Oxygen was determined by Winkler's method, while salinity was calculated by the amount of Cl (determined by $AgNO_3$).

Only an insignificant variation in the oxygen value, from 0.2 to 0.3, was permitted in any of the experiments.

The experiments were conducted in crystallizers having a capacity of 500 cm³ into which were poured 300 cm³ of sea water solution, prepared as described above, and 20 eggs were placed in it. Controls were prepared with pure sea water. A calculation of the living embryos and of the larvae hatched, as also of the percentage of dead fish and eggs, was made 15, 39 and 52 hours after the beginning of the experiment; the reaction of the solution was also tested.

The control experiments under laboratory conditions gave from 95 to 100% positive results, which numerical interval must be considered as 100%, since, for unknown reasons, there were defective eggs which died after 10—15 hours.

I n f l u e n c e o f T e m p e r a t u r e. Special experiments with respect to the influence of temperature were not carried out as a result of the station's lack of equipment for that sort of work.

All experiments concerning salinity or the influence of the active reaction were conducted in a basement, where the temperature was fairly even, fluctuating between 20 and 22° C. Such temperatures as these were observed in the upper layers of the sea water during June and July. When the sea water is heated more than this, the eggs, as is known, sink to a deeper layer.

A rise of temperature above 18 to 21°C greatly hastens the development of the embryo. Preliminary observations indicate that 26° C is the upper limit of temperature.

At a temperature of 15° C development stops. If eggs be subjected to 15° C for 5 hours and then abruptly transferred to 21—22°C a large percentage of deaths is observed.

Salinity. The salinity of the Azof Sea is 9—10‰. In some parts, freshened by the inflow of rivers, the salinity drops, as, for example at Achnyev, it falls to 6.28‰.

Table 1

Influence of salinity on the eggs of *Engraulis encrasicholus maeoticus*

	Salinity ‰	Hatch of larvae 39 hours after beginning of experiment in%	Hatch of larvae 52 hours after beginning of experiment in %	Remarks
16 blastomeres				
1. pH 8.17 . . .	5.364	0	0	Eggs turn pale after 52 hours and die
2. pH 8.42 . . .	10.098	97	97	
32 blastomeres				
1. pH 8.10 . . .	6.30	51.5		Eggs and larve lie on the bottom.
2. pH 8.37 . . .	15.138	97		Black Sea water accelerates hatching and death.
3. pH 8.12 . . .	17.334	100		Eggs and larvae float on the surface.
4. pH 8.10 . . .	19.44	90		
120 blastomeres				
1. pH 8.12 . . .	5.320	70		Eggs and larvae lie on the bottom.
2. pH 8.12 . . .	26.28	50	100	Larvae are crooked, being hatched they die and are drawn by the surface

From the data obtained, the conclusion may be drawn that a salinity of 5.36‰ is the limit for the stage of 16 blastomeres since there was no hatching of larvae. With a salinity of 6.3‰ only 50% hatched, the larvae lay on the bottom and felt ulwell.

L. Woyrley, working with mackerel roe, considered hatchings of 50% as negative.

In records which have come to me from Academician N. M. Knipovich it is stated that near Achuyev there were many anchovy larvae and eggs in the catch, but that many of them were dead.

The data of the experiment agree with what is observed in nature. Larvae and eggs, falling into a medium with a salinity lower than normal, die. In the records, too, an interesting phenomenon is noted—larvae and eggs were taken here at a depth of $3^1/_2$ m and lower on the bottom; whereas, ordinarily, eggs do not sink so low. It may be that here the difference in specific gravity of the water and the eggs had its influence. In water with a salinity raised to 15‰, in experiments with 32 blastomeres, development of embryos and hatching of larvae is hastened.

In water having a salinity of 19.4‰—that is equal to the salinity of Black Sea water, the eggs are suspended at the very surface, the larvae hatch, but quickly die, adhering to the surface of the water and literally dissolving under one's eyes.

The calculation, in such cases, has to be made on the basis of the empty egg envelopes.

In water with a salinity higher than that of the Black Sea but lower than that of the Mediterranean, that is 26‰, the hatch of larvae is much delayed, the latter being deformed and non viable.

These experiments show that the Azof form is substantially individualized from the Black Sea form in its physiological development It is characteristic that when the salinity of the water was higher than usual in the Azof Sea, due to a rush of the Black Sea water, many dying eggs were found in samples from the sea, while in the experiments, although there was observed an accelerated development, there was also noted a high percentage of deaths.

Engraulis encrasicholus ponticus. The salinity of the Black Sea is 16.6—19‰.

Table 2

Influence of salinity on the eggs of *Engraulis encrasicholus ponticus*

	Salinity ‰	Percentage of larvae hatched 39 hours after beginning of experiment	Percentage of larvae hatched 52 hours after beginning of experiment	Remarks
32 blastomeres				
1. pH 8.25	8.82	15	20	Larvae die soon
2. pH 8.25	10.80	79	93	Larvae in good condition
3. pH 8.08	10.4	97	97	
4. pH 7.45	21.24	95	95	
5. pH 8.25	21.9	97.5	97.5	47% die
6. pH 8.25	33.5	100.0	100.0	46% die
Control	16.65	97.5	97.5	6.2% die

As may be seen from the Table 2, experiments were conducted with the salinity of the Azof Sea, the Mediterranean, and, as a control, Black Sea water. The hatch of larvae in water with a salinity equal to that of Azof Sea water (10.4‰ and pH=8.08) is good, and some of them live a comparatively long time, but 30‰ die; whereas, in the control, the death-rate is from 3 to 6%. In water having a 33‰ concentration, hatching is greatly accelerated, and 100% hatch but the larvae are deformed and non viable, the death-rate is 46%.

The embryos proved to be sensitive not only to water having a salinity of 33‰, but also to water with a lower salinity. Thus 21.9‰ gave a high death-rate.

The experiments show that although the Black Sea variety hatches well in the salinity of the Azof Sea, still, the best medium for its development is a salinity of 16.05‰, as is shown by the death-rate of larvae. Unfortunately, owing to the highly frequent North-East winds, at Novorossiisk but little *Engraulis* eggs could be catched, and it was not possible to conduct experiments with low salinities.

The influence of active reaction. The active reaction of the upper layers of Azof Sea water is pH=8.1.

The active reaction of Black Sea water is pH=8.25.

The eggs of *Engraulis encrasicholus* of both subspecies were brought into the laboratory and put into solutions of sea water having different pH,

but with a salinity for *Engraulis maeoticus* equal to that of the Azof Sea water and for *Engraulis ponticus* to that of the Black Sea one.

Table 3

The Influence of the Active Reaction on the Eggs
(Hatch of larvae in %)

Number of blastomeres \ pH	6.56	6.92	7.04	7.78	8.08	8.17	8.27	8.47	8.87	
	Engraulis encrasicholus maeoticus									Reckoned after 39 h. from the beginning of the experiment.
64	Death	13%	44%	75%	95%	97%	97.5%	93.5%	45%	
	Enraulis encrasicholus ponticus									
64		5%	—	—	77.5%	—	96.7%	92.5%	50%	
100	Death		75%							After 64 h.

In sea water having a pH =5.12, there occurs a 100% death of the eggs after two hours, both of *Engraulis encrasicholus ponticus* and *maeoticus*. In water having pH =6.56, the death of the eggs begins after 5 hours, in water with pH =6.92 in the case of the Azof Sea form, the death-rate is 33% after 18 hours, and after 42 hours 13% hatch, but the larvae die soon. Only 5% of the eggs of *Engraulis encrasicholus ponticus* hatch out.

In neutral solutions of pH =7.04—7.06 the hatching is delayed, and after 64 hours, *maeoticus* hatched 85% and *ponticus*, 75%, whereas in the control, these percentages were obtained after 39 hours.

When the alkalinity is raised to pH =8.4—8.5, an acceleration of development in both forms, amounting to 20—25%, is observed.

When the alkalinity is raised to 8.7, the percentage of hatch is lowered in both subspecies.

With pH =8.8—9.1, hatching is greatly retarded in the case of the *ponticus*, and the larvae are deformed and die on emerging.

In the case of the Black Sea form of *Engraulis encrasicholus* with normal salinity and pH =8.28, hatching occurs after 39 hours, just the same as in the control.

When the alkalinity is lowered to that of the Azof Sea (8.08), normal larvae hatch, with some retardation, however, and the percentage of deaths was in our experiments increased.

Conclusions

1. The limit of salinity for *Engraulis encrasicholus maeoticus* is 5.36‰ for the stage of 16 blastomeres. In water with a salinity equal to that of the Black Sea, the embryos are deformed and die soon.

The eggs of *Engraulis encrasicholus ponticus* are capable of developing within the limits of 8 to 21‰. If the salinity be brought to that of the Mediterranean, where this genus also lives, the embryos of *Engraulis encrasicholus ponticus* are deformed and die soon.

2. The pH limit, on the side of acidity, for *Engraulis encrasicholus maeoticus* is 6.52.

The eggs of *Engraulis encrasicholus ponticus* are capable of developing within the range of pH =6.92 to pH =8.8 or 8.9.

The rise of pH up to 8.4—8.5 hastens the development of the embryos.

3. A slight rise of the salinity of the medium, as also of its pH above normal conditions hastens the development of the embryos and acts as a stimulating agent.

4. The eggs of *Engraulis encrasicholus maeoticus* from the Azof Sea and those of *Engraulis encrasicholus ponticus* manifest varying sensitivities to changes in the salinity as well as of the active reaction of the medium, which argues in favour of the assumption that these are independent genotypical forms.

In conclusion, I desire to extend thanks to S. N. Skadovksi for the advice, he so kindly extended me in my work and to V. A. Vodyanitski for having provided me with facilities for my work at hydrological station.

Received
2. III. 1935

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